Gas Dynamic Gain of Supersonic Thrust Nozzles

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Thrust misalignment in rocket nozzles is investigated for the situation where the gas flowfield is misaligned with respect to the axis of the supersonic region of the nozzle. The analysis is based upon the three-dimensional method of characteristics, and hence considers the full nonlinearity of the flowfield. Numerical results are presented for a wide range of nozzle geometries. These results yield considerable insight into the thrust misalignment phenomenon, and indicate means by which thrust misalignment arising from certain kinds of nozzle misalignment can be reduced. Implications to thrust vector control are also discussed.

Nomenclature

a = speed of sound
F_x, F_y = axial and side thrust component, respectively
G = gas dynamic gain, δ_e/δ
H = stagnation enthalpy
I_{F_x} = axial specific impulse, $F_x/\dot{m}$
I_{F_y} = side specific impulse, $F_y/\dot{m}$
I_{M_z} = specific moment, $M_z/\dot{m}y_t$
l = distance from throat center to missile mass center
$\dot{m}$ = nozzle mass flow rate
M_i = Mach number at initial-value surface
M_z = moment about z axis
p, P = static and stagnation pressure, respectively
R = gas constant
T = stagnation temperature
u, v, w = velocity components
x = axial coordinate, or nozzle length
y = lateral coordinate
z = axis of moment
y_t = throat radius
α = nozzle cone half angle
γ = specific heat ratio
δ = nozzle misalignment angle
δ_e = effective thrust misalignment angle
ε = nozzle expansion ratio
θ_i = gas misalignment angle on initial-value surface
λ = throat curvature ratio, ρ_t/y_t
ρ = density
ρ_t = nozzle throat circular arc radius of curvature

Introduction

THRUST misalignment is that condition in which the propulsive thrust vector does not intersect the missile center of mass. This misalignment results in turning moments about the missile mass center which causes the missile to roll, pitch, or yaw, and creates a lateral force component which causes the missile to translate. Of these two effects, the turning moments about the missile mass center are the major contributors to excursions from the intended flight path. Even the lowest levels of thrust misalignment presently obtainable cannot be tolerated in free-flight rocket applications, and missile spin must be introduced to alleviate the resulting dispersion. In certain types of missile guidance schemes, thrust misalignment compromises gain settings and again results in flight path errors. Thrust misalignment is thus of considerable importance, and its reduction is a continuing goal in research and development programs.

Of the many sources of thrust misalignment, two may be attributed to the rocket motor: irregularities in the nozzle geometry, and irregularities in the gas flow entering the nozzle. The present paper is concerned with thrust misalignment caused by misalignments between the gas flowfield and the supersonic section of the nozzle.

The siguation investigated here was also considered by Darwell and Trubridge.[1] Their theoretical work is based on a perturbation analysis of one-dimensional, isentropic nozzle flow. The perturbation consists of two parts: a symmetrical effect caused by departures of axisymmetric flow from one-dimensional conditions, and an asymmetrical effect caused by the disturbance. Results are presented for a range of conical nozzle geometries.

The present work is similar to that of Darwell and Trubridge in scope; however, the flowfield analysis is based upon the three-dimensional method of characteristics program developed by Ransom, Thompson, and Hoffman,[2] as opposed to the linearized method employed by Darwell and Trubridge. By employing a numerical technique based on the method of characteristics, the complete nonlinear governing equations are solved. Thus, no restrictions need to be placed on the size of the disturbance, its form or the shape of the supersonic nozzle contour. The situations which can be analyzed are limited only by one's ability to described accurately the initial disturbance itself.

Specifically, this paper presents the results of a parametric study of thrust misalignment in conical nozzles caused by an angular misalignment of the flowfield in the throat of the nozzle with respect to the axis of the supersonic nozzle section. The effects of misalignment angle, throat radius of curvature, cone angle, and nozzle length are investigated. Some results are given for contoured nozzles designed by Rao's technique.[3]

Flowfield Analysis

The numerical results presented here were obtained with the computer program developed by Ransom, Thompson, and Hoffman.[2] A brief description of that program follows.

The continuity equation, the Euler momentum equation, the equation for entropy conservation along streamlines, and the fluid equation of state were applied to the inviscid, steady, adiabatic flow of a gas having frozen (present study) or equilibrium chemical composition in a three-dimensional nozzle. Those equations are

$$\rho_t + \nabla\cdot(\rho\bar{V}) = 0 \qquad \text{(continuity)} \qquad (1)$$

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$$\rho(D\bar{V}/Dt)+\nabla p = 0 \qquad \text{(Euler equation)} \tag{2}$$

$$Dp/Dt - a^2(D\rho/Dt) = 0 \qquad \text{(isentropic flow)} \tag{3}$$

$$a = a(p, P, H) \qquad \text{(equation of state)} \tag{4}$$

The existence of shock waves as discrete discontinuities is excluded. Gradients in entropy and stagnation enthalpy normal to the streamlines are permitted.

Equations (1–4) comprise a system of quasi-linear, hyperbolic, partial differential equations with the three space coordinates as independent variables. The hyperbolic property permits the use of a marching type integration scheme in the flow-wise direction, since each point in the flow has a finite zone of dependence.

Ransom et al.[2] developed a computer program employing a second-order bicharacteristics method for solving the above set of equations for three-dimensional supersonic nozzle flow. The flow is permitted to have up to eight planes of symmetry. One sector of the flow is calculated, and the remaining sectors are determined by reflection about the planes of symmetry. In the present study, one plane of symmetry is assumed.

Much of the character of a flow is determined by the conditions specified on the initial-value surface. For example, whether or not gradients in stagnation pressure and enthalpy exist depends upon whether gradients in these properties exist on the initial-value surface. The initial-value data consist of the specification of the velocity components, pressure, stagnation pressure, and stagnation enthalpy over a planar surface normal to the x-coordinate direction. For the present investigation, the initial-value surface is a constant property plane pitched at an angle with respect to the axis of an axisymmetric nozzle exit contour.

In many propulsive nozzles, shock waves occur in the supersonic portion of the flowfield. Generally, those shock waves are relatively weak, resulting in no significant increase in the entropy of the fluid. The present algorithm does not consider shock waves as discrete discontinuities. Shock waves are smeared out over several grid points, but experience has shown that approach to be quite accurate, even for relatively strong shock waves. For the range of gas misalignment considered in the present study (up to 2°), the present method should account for the effects of shock waves quite adequately.

The problem solution comprises velocity components and pressure at each of the intersections of a selected system of streamlines with surfaces perpendicular to the x axis. A six-component thrust integration is performed at each solution surface, which yields the three force components and the three couples. The origin of the coordinate system is the central point at the nozzle throat.

The numerical algorithm has been programed for nozzle flows for the CDC 6500 computer using the FORTRAN IV language.‡ Various options permit the solution of a wide range of exhaust nozzle problems.

Thrust Misalignment Study

There are many sources of gas misalignment in the supersonic section of a nozzle, for example: nonuniform combustor conditions, asymmetric entrance, throat, or exit geometry, and a misalignment of the symmetric supersonic contour with respect to the axis of the missile. The last effect, supersonic contour misalignment, is used to advantage as a thrust vector control mechanism, which is the principle of the split-line nozzle concept. Any of those sources of gas, and hence thrust, misalignment can be determined by the computer program discussed in the previous section if the properties on the initial-value plane can be specified.

The present study is concerned only with thrust misalignment due to an angular gas flowfield misalignment with respect to the supersonic nozzle contour. The results given in the following sections can be employed to estimate the thrust misalignment due to an undesired geometric misalignment, or to estimate the thrust vector control attainable by a sonic split-line nozzle at small deflection angles.

‡ The program is available from the Aero Propulsion Lab., Ramjet Technology Branch, Wright-Patterson Air Force Base, Ohio.

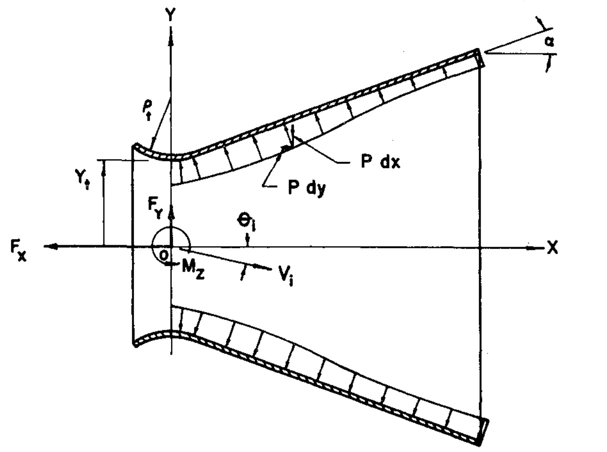

Fig. 1 Nozzle geometry and force model.

The assumption, employed in the present analysis, that the transonic flowfield is a uniform flow pitched at an angle with respect to the geometric axis of the nozzle is a simplication of the actual transonic flowfield in a converging-diverging nozzle,[4] even when the entering flow is symmetrical. If gas misalignment occurs in the converging portion of the nozzle, the resulting flow distortion on the supersonic initial-value plane must be accounted for accurately if precise thrust misalignment results are to be obtained. For that case, the results presented in the present paper are only qualitatively representative of the thrust misalignment in the actual case. However, for the case where the diverging portion of the nozzle is geometrically misaligned, the results presented herein are quantitatively accurate for nozzles having a large throat radius of curvature so that the sonic surface approaches a plane. For nozzles having a small throat radius of curvature, the results are qualitative in nature. In any case where accurate initial-value data are known, they may be employed in the program to determine quantitatively accurate thrust misalignment results.

The geometric model considered is illustrated in Fig. 1. The geometric parameters of interest in a conical thrust nozzle are y_t, ρ_t, α, and x. The performance parameters of concern are $\dot{m}$, F_x, F_y, and M_z. Since only angular misalignments are considered here, the pressure distribution about the plane shown in Fig. 1 is symmetric, and thus no other forces are present, that is, $F_z = M_x = M_y = 0$. The actual force acting on the nozzle is obtained by adding the force developed at the throat section (that is, the momentum and pressure forces) to the pressure forces applied to the supersonic portion of the nozzle. The line of action of this force will not pass through the origin of the coordinate system, located at the throat section, when gas misalignment is present. It is convenient to specify the actual force in terms of its two components, F_x and F_y, acting at the origin as illustrated in Fig. 1, and the couple M_z. The couple M_z is introduced for the sole purpose of translating the thrust components to the geometric nozzle throat center.

The nozzle operating conditions are specified by γ, R, P, and T. The properties on the initial-value plane are completely determined by specifying the Mach number of the uniform flow M_i, which was chosen to be 1.10. The gas misalignment is specified by the pitch angle θ_i.

When values of y_t, ρ_t, α, and nozzle length x are specified, the performance parameters $\dot{m}$, F_x, F_y, and M_z can be evaluated by the three-dimensional method of characteristics program described above. The flow is isentropic, and certain scaling laws are applicable. Since all mass flow rates, forces, and couples scale with nozzle size, the results are presented in the form of specific impulse and specific moment. Thus

$$I_{F_x} = F_x/\dot{m} \qquad \text{(lbf-sec/lbm)} \tag{5}$$

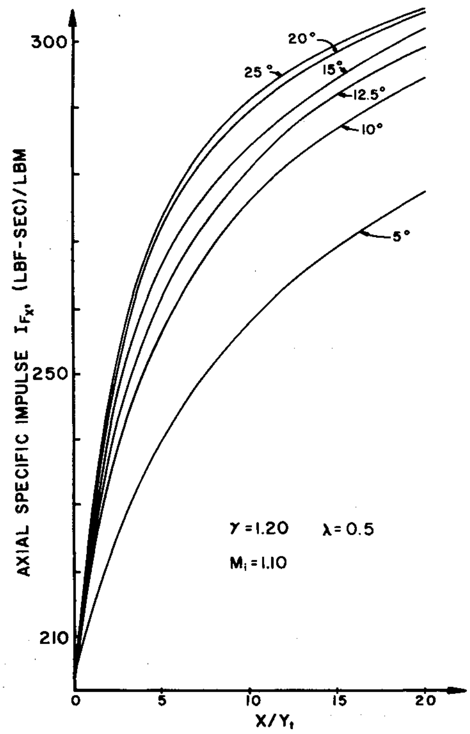

Fig. 2 Effect of cone angle on axial specific impulse.

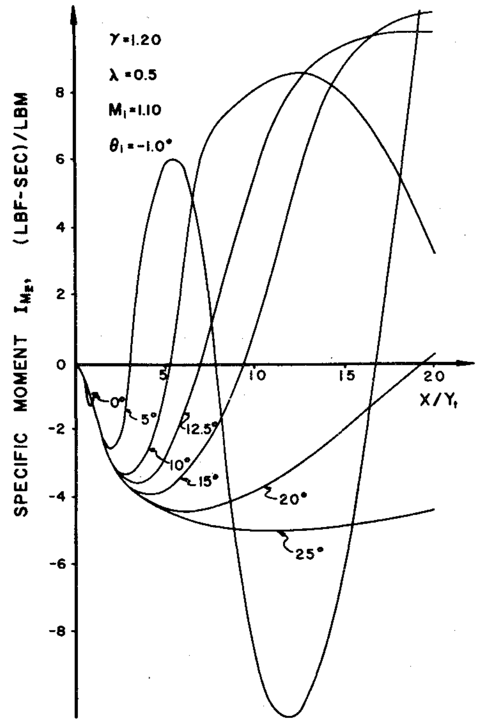

Fig. 4 Effect of cone angle on specific moment.

$$I_{F_y} = F_y/\dot{m} \quad \text{(lbf-sec/lbm)} \tag{6}$$

$$I_{M_z} = M_z/\dot{m}y_t \quad \text{(lbf-sec/lbm)} \tag{7}$$

For isentropic nozzle flows discharging into a vacuum, the specific impulse and specific moment are independent of stagnation pressure and vary directly with $(RT)^{1/2}$. No scaling is possible for different values of specific heat ratio γ. The nozzle throat geometry is completely specified by the ratio of the throat radius of curvature ρ_t to the throat radius y_t, denoted by λ. The results presented are for gas properties of $\gamma = 1.2$, $R = 60$ (ft-lbf)/(lbm-°R), $P = 1000$ psia, $T = 6000$°R, and a vacuum back pressure. Through the use of the scaling laws, the results presented herein can be extended to encompass a broad range of operating conditions.

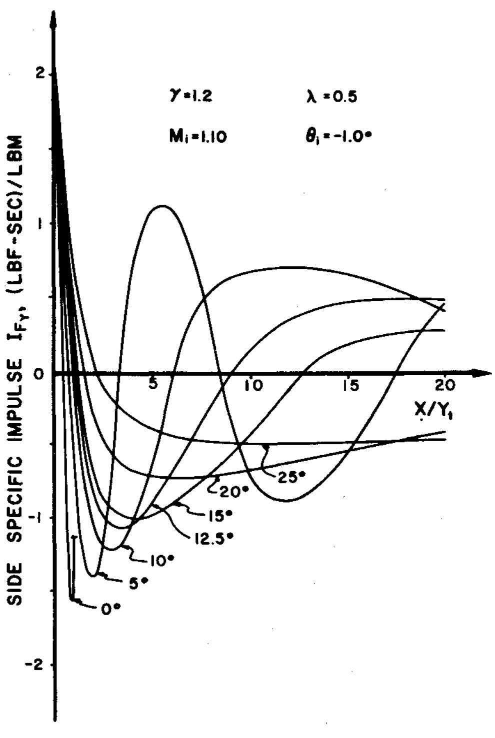

Fig. 3 Effect of cone angle on side specific impulse.

Results

Figure 2 presents the axial vacuum specific impulse for a range of cone angles and zero pitch angle (no misalignment). These values may be considered exact for the range of pitch angles of interest since the axial component of thrust will vary less than 0.1% for pitch angles as large as 2°. The axial thrust results are quite typical, and they are presented solely for use in a later section concerned with designing conical nozzles to minimize thrust misalignment. These results were obtained with the three-dimensional, method of characteristics program, and thus include nozzle divergence losses.

Figures 3 and 4 present the side specific impulse and specific moment for a family of conical nozzles having a throat radius of curvature ratio of $\lambda = 0.5$ and cone angles from 0° to 25°, for a pitch angle of $-1.0°$. The oscillatory nature of these results, predicted by Darwell and Trubridge,[1] is obvious. The disturbances are basically pressure waves, which propagate through the flowfield along the Mach conoids of the flow. Thus, the initial disturbance propagates from side to side across the nozzle as it simultaneously travels downstream. A net momentum transfer to the fluid occurs as the pressure waves traverse each fluid element across the nozzle, and the magnitude of the pressure disturbance is decreased with each reflection. This effect decreases the magnitude of the side force as the flow proceeds down the nozzle, as illustrated in Fig. 3. However, as the distance from the nozzle throat increases, the couple increases in magnitude because of the increasing moment arm of the side force, as illustrated in Fig. 4.

In nozzles having smaller cone angles, the rate of expansion is smaller, and the Mach number remains at a smaller value at the same axial position than in a nozzle with a larger cone angle. Thus, the Mach angle is larger, the Mach conoids have a steeper slope, and the pressure waves travel back and forth across the nozzle more rapidly. As the cone angle is increased,

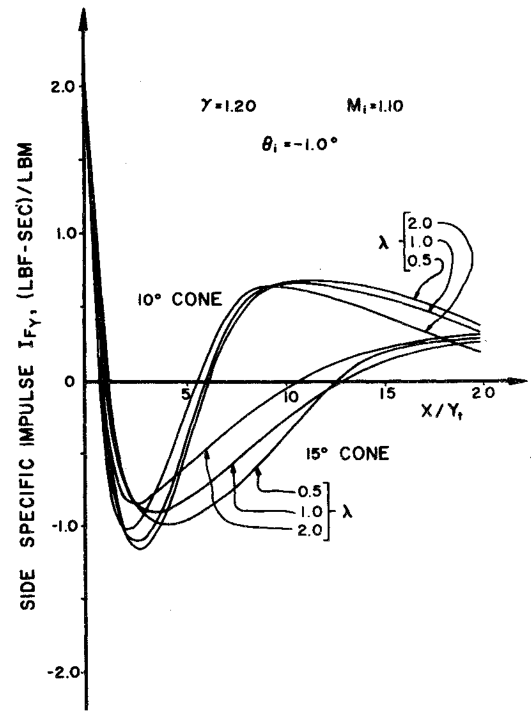

Fig. 5 Effect of throat radius of curvature on side specific impulse.

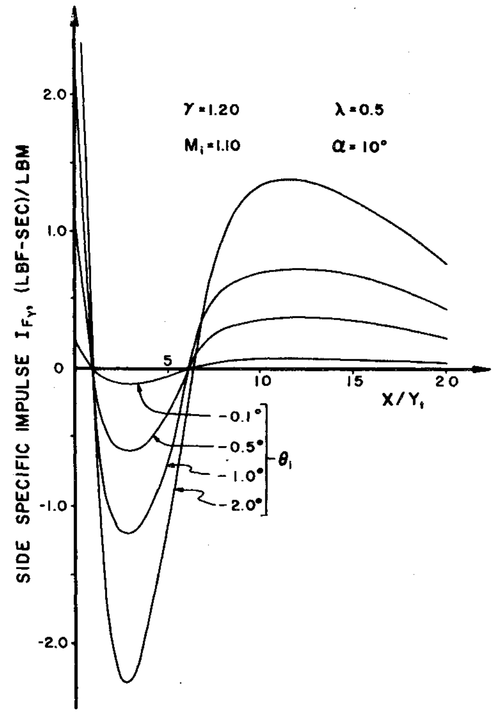

Fig. 7 Effect of disturbance size on specific impulse.

the flow expands more rapidly, which results in larger Mach numbers, smaller Mach angles, and Mach conoids of decreased slope. Thus, the frequency of oscillation of both the side force and the couple should decrease as the cone angle increases. All of the aforementioned effects would be anticipated based solely on the general features of supersonic flows. Nor surprisingly, all of these trends are observed in Figs. 3 and 4. Thus, the items of interest in Figs. 3 and 4 are the exact magnitudes of the forces and moments and their rate of oscillation.

The effect of the throat radius of curvature ratio λ is illustrated in Figs. 5 and 6 for the 10° and 15° conical nozzles. The effect of this parameter appears to be slightly more pronounced at the larger cone angle. The magnitudes of both the forces and the couples are altered, as are the shapes of the curves. However, the cone angle itself is seen to have a much larger influence than the throat radius of curvature ratio. In general, as the throat radius of curvature ratio increases, the expansion takes place more slowly and the pressure disturbances reflect more rapidly for the same reasons as discussed in the cone angle studies

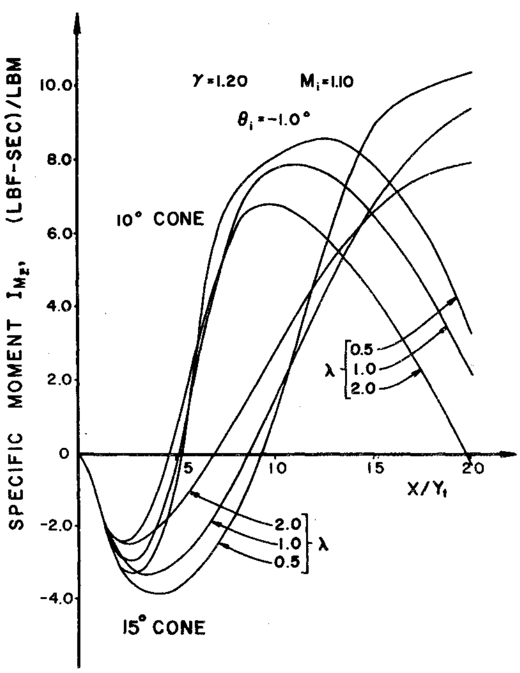

Fig. 6 Effect of throat radius of curvature on specific moment.

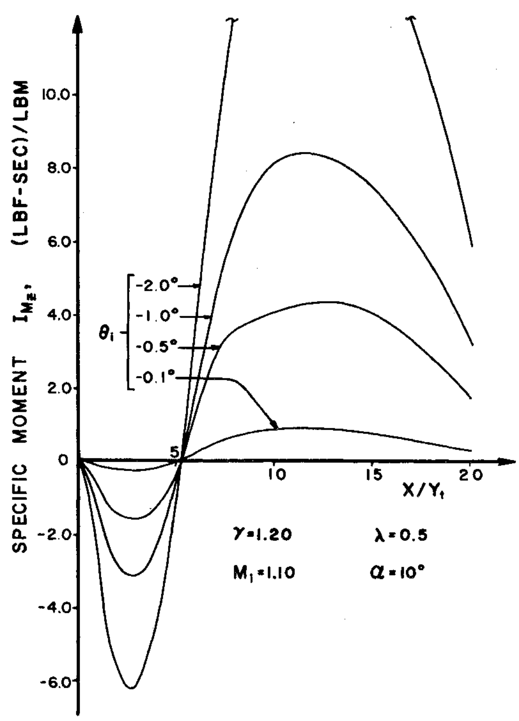

Fig. 8 Effect of disturbance size on specific moment.

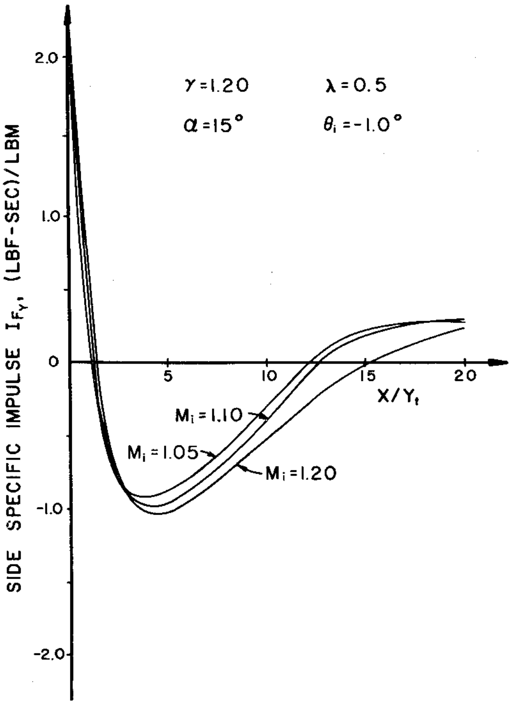

Fig. 9 Effect of initial Mach number on specific impulse.

described previously. Thus, the side forces and moments oscillate more rapidly when the radius of curvature ratio is increased.

Figures 7 and 8 present the side forces and couples in a 10° conical nozzle for values of the pitch angle θ_i from $-0.1°$ to $-2.0°$. From these results it is seen that the side forces and couples scale directly with the pitch angle in this range of the pitch angle. A slight shift in the second zero of the side force for the $-2.0°$ pitch angle is observed, and it seems that some nonlinear scaling may be on the verge of occurring. Hence, the results presented herein should not be scaled beyond a pitch angle of $-2.0°$ without further analysis. Within that limit, however, it appears quite valid to scale all the side forces and moments with the pitch angle. Thus, the ordinates of Figs. 3–6 may be regarded as having the values indicated per degree of pitch.

The uniform flow initial-value plane employed in these parametric studies is obviously a simplification. As shown by the studies of Back, Cuffel, and Massier,[4] the flow in the throat region of a converging-diverging nozzle is certainly nonuniform. The sensitivity of the results to the initial Mach number is illustrated in Figs. 9 and 10. While these results are also subject to the errors of the uniform flow assumption, they do indicate some general effects of the throat properties on the side forces and couples. The major effect of the initial Mach number on the side force appears to be in the rate of oscillation of the side force. This effect is primarily due to the effect of Mach number on the slope of the Mach lines. The couples also oscillate at different rates. In addition, the maximum magnitude of the couple increases as the initial Mach number increases. This effect appears to be due primarily to the displacement of the side force rather than to a change in magnitude of the side force.

Darwell and Trubridge[1] suggested that gas misalignment effects can be reduced through dampening by a short parallel section at the nozzle throat. Figure 11 presents the side specific impulse in a cylindrical section for values of the initial Mach number of 1.05, 1.10, and 1.20. The coupling between rate of oscillation and Mach number is very apparent in these results. However, the magnitude of the side force decays only slightly, approximately 15% on the first cycle and considerably less on the succeeding cycles. These results do not support the assertion of Darwell and Trubridge regarding the effect of an elongated throat section, and no explanation for this difference of opinion was found during the present investigation.

In Fig. 12, some results for contoured nozzles designed by Rao's technique[3] for a vacuum are presented. The Rao nozzle contours are also presented in Fig. 12. These results are quite different from those obtained for conical nozzles having small cone angles, but the behavior is quite similar to that of the larger cone angle nozzles. This is not surprising, since Rao nozzles open up to a rather large initial expansion angle and then turn the flow back with a very gradual contour. The effect of throat radius of curvature on side force is similar to that observed for conical nozzles.

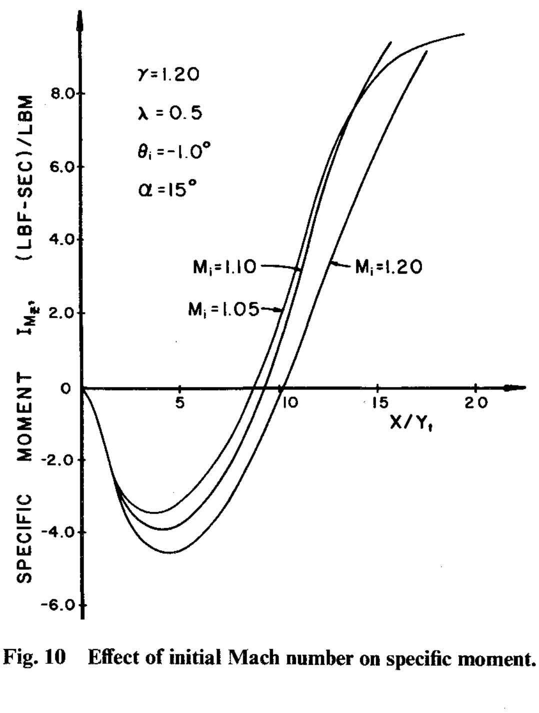

Fig. 10 Effect of initial Mach number on specific moment.

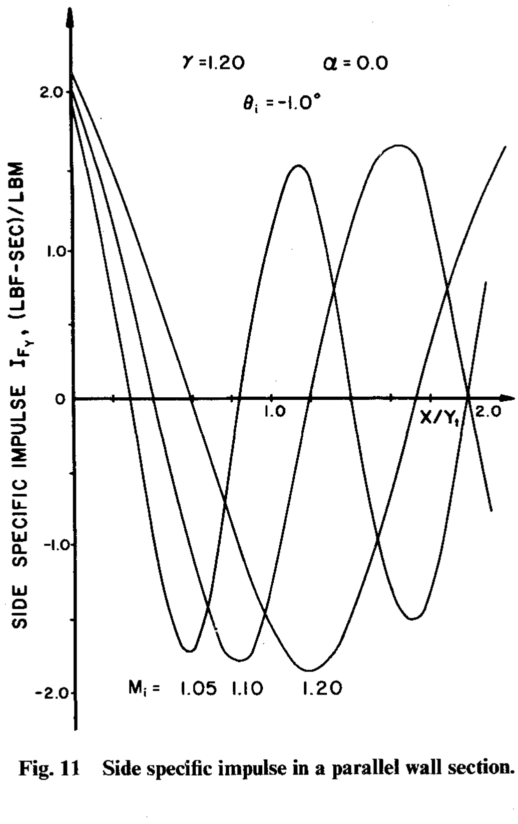

Fig. 11 Side specific impulse in a parallel wall section.

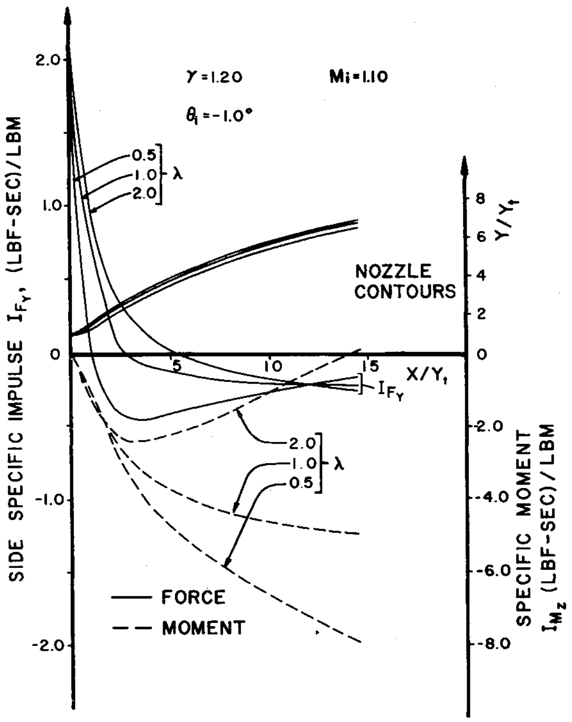

Fig. 12 Results for Rao nozzles.

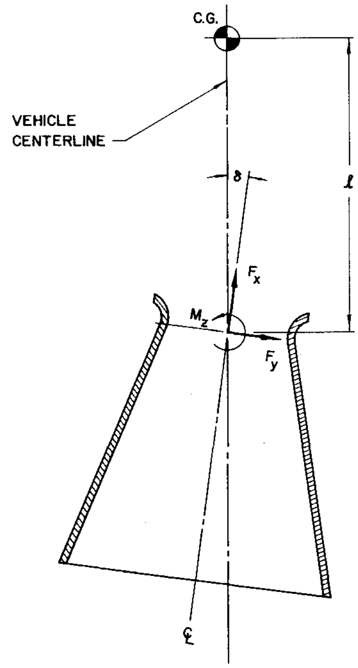

Fig. 14 System geometry for misaligned nozzle.

Comparison with Previous Results

In this section, a brief comparison is made between the present method and the method of Darwell and Trubridge.[1] In Fig. 3 of their paper, Darwell and Trubridge present the cone angle α as a function of the ratio of nozzle exit diameter to throat diameter D/D_t required to yield zero lateral force at the nozzle exit. Their results show the second and third zeroes of the lateral force variation. Figure 13 reproduces a portion of their results, along with comparable results obtained by the present method. The upper curves pertain to the second zero, and the lower curves pertain to the third zero. The two sets of results are in fair qualitative agreement, although the quantitative agreement is not very good. It is not possible from their paper to determine the exact form of the asymmetry. Hence, the results in Fig. 13 should be taken as a casual comparison of the two methods, and not interpreted as a rigorous quantitative comparison.

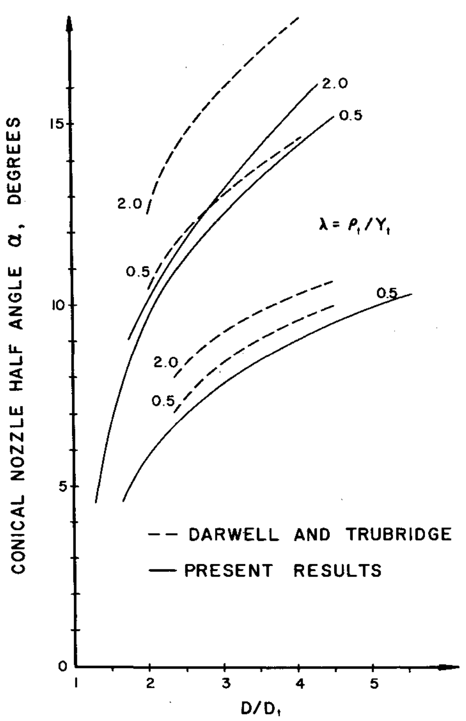

Fig. 13 Comparison with results of Darwell and Trubridge.

Application to Nozzle Design

The preceding results have important implications to nozzle design. In their investigations, Darwell and Trubridge[1] have shown that nozzles can be designed to eliminate the effects of a misaligned gas flowfield entering an aligned nozzle. Since the choice of the reference defining the misalignment may be either the entering gas flowfield or the nozzle axis, the previous figures are valid regardless of whether the source of the misalignment is the flowfield or the supersonic nozzle contour. Thus Figs. 2–6 can be used to select nozzle parameters that reduce the effects of misaligned nozzle exit cones.

Figure 14 depicts a supersonic nozzle misaligned with respect to the missile axis. Summing moments about the missile mass center and making the small angle approximations lead to

$$\Sigma M_{CG} = M_z + lF_y + lF_x\delta \tag{8}$$

where δ is measured in radians. If an "effective" thrust misalignment angle δ_e is defined as

$$\delta_e = \Sigma M_{CG}/lF_x \tag{9}$$

and definitions (5–7) are introduced, then Eq. (8) may be written as

$$\frac{\delta_e}{\delta} = 1 + \frac{1}{I_{F_x}}\left[\frac{(I_{M_z}/\delta)}{(l/y_t)} + (I_{F_y}/\delta)\right] \tag{10}$$

The ratio δ_e/δ is the degree of thrust misalignment per degree of nozzle misalignment, and as such represents the gas dynamic gain of the nozzle. It will be recalled that the side specific impulse I_{F_y} and the specific moment I_{M_z} presented in Figs. 3–6 may be regarded as being per degree of misalignment (at least up to 2°). If this convention is adopted, and if the ratio δ_e/δ is defined to be the gas dynamic gain G, then Eq. (10) becomes

$$G = 1 + \frac{(180/\pi)}{I_{F_x}}\left[\frac{I_{M_z}}{(l/y_t)} + I_{F_y}\right] \tag{11}$$

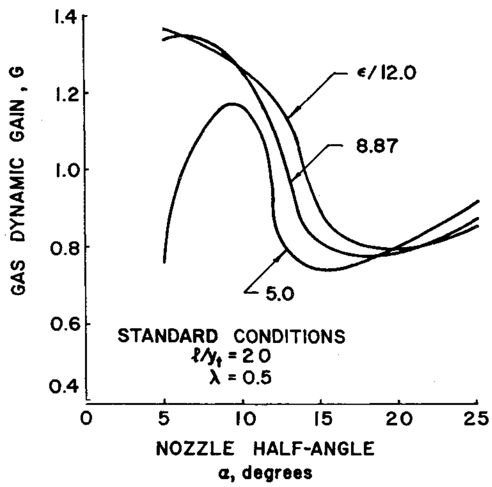

Fig. 15 Gain for a family of typical sea level nozzles.

As a consequence of the linear behavior of the side specific impulse and specific moment with misalignment angle, the gain equation is free of any dependence on δ.

The gas dynamic gain for a family of conical nozzles can be characterized through the use of Eq. (11) and Figs. 2–6. To use these figures it is only necessary to express the nondimensional nozzle length, x/y_t, in terms of the nozzle geometric parameters α, λ, and ε as follows:

$$x/y_t = \lambda \sin\alpha + [(\varepsilon)^{1/2} - 1 - \lambda(1-\cos\alpha)]\cot\alpha \qquad (12)$$

The gain curve for a family of conical nozzles having a throat radius of curvature ratio λ of 0.5 is presented in Fig. 15. The ratio of the distance to missile mass center to the throat radius was fixed at a value typical of a single-stage free-flight rocket (i.e., $\varepsilon = 20$), and the axial vacuum specific impulse of Fig. 2 was corrected to standard conditions (1000 psia chamber pressure and sea level ambient pressure) for use in constructing the gain curve. For the case of optimum expansion ($\varepsilon = 8.87$), a minimum gain of $G = 0.775$ is encountered at an 18° half-angle. It is therefore possible to reduce the effect of nozzle geometry misalignment on thrust misalignment by 22.5%. While some latitude does exist in the selection of nozzle parameters that minimize gain, in general there will be a tradeoff with specific impulse performance and nozzle length. Similarly, it is possible to select nozzle parameters that would amplify the effect of geometry misalignment—by approximately 35% for the case of optimum expansion for the example in Fig. 15. Thus the potential exists to enhance gas dynamically the thrust vectoring obtained from nozzle vectoring. An analysis such as that reported here must be conducted at higher misalignment angles, however, to fully characterize this design potential.

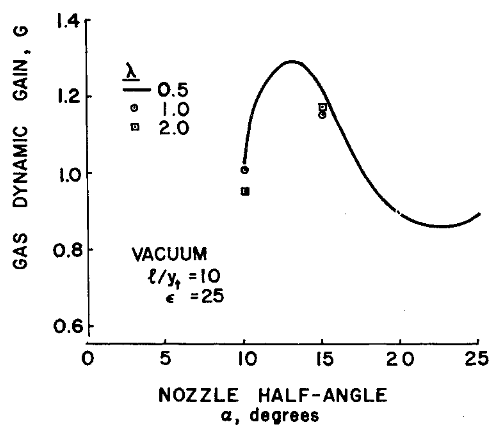

Fig. 16 Gain for a family of typical high altitude nozzles.

Figure 16 presents the gas dynamic gain for a family of nozzles that might be typical in an upper stage space application. As before, both maximum and minimum gain points exist, but the latter does not occur at a nozzle half-angle that would be reasonable for design purposes. Only limited computations of side specific impulse and specific moment are available for throat radius of curvature values other than 0.5; therefore, only the trend is indicated by the few points plotted. It appears that the throat radius of curvature ratio λ does not have a pronounced effect on the location of the gain extrema. Figure 11 does imply, however, that some additional shifting of the gain curve is possible through the use of a short parallel section at the nozzle throat.

It will be recalled that Darwell and Trubridge[1] analyzed the situation in which the nozzle is aligned, but subject to a misaligned gas inlet condition. If the nozzle misalignment is set to zero (i.e., $\delta = 0$) in Eq. (8), then the gas dynamic gain for the Darwell-Trubridge situation becomes

$$G = \frac{(180/\pi)}{I_{F_x}}\left[\frac{I_{M_z}}{(l/y_t)} + I_{F_y}\right] \qquad (13)$$

This differs from the more general condition, Eq. (11), only by the additive constant 1.0. The terms I_{F_y} and I_{M_z} are here interpreted as being per degree of entering gas flowfield misalignment. Figures 15 and 16 may thus be used for the Darwell-Trubridge situation by subtracting 1.0 from the ordinate scale. For the Darwell-Trubridge situation, it is possible to eliminate the effects of gas misalignment altogether, since design conditions in which $G = 0$ can be achieved. (As previously discussed, there is some quantitative disagreement between the zeroes predicted by the present analysis and those reported by Darwell and Trubridge.) A nozzle designed for the Darwell-Trubridge situation would neither dampen nor amplify the effect of nozzle geometric misalignment, but would insure that the thrust axis coincided with the nozzle axis. A nozzle designed to minimize the effects of nozzle geometric misalignment would reverse the sign and dampen but not nullify the response to entering gas misalignment. Thus if both types of misalignment are present, their relative magnitudes must be known before their combined influence can be minimized.

Conclusions

A method for analyzing the effects of gas misalignment on nozzle thrust misalignment was presented. The procedure is readily applicable and available in the form of a production type computer program. Extensive numerical results were shown for a family of conical nozzles. These results serve both to illustrate the procedure and to provide a quantitative understanding of the effect of gas misalignment on thrust misalignment. In a real situation, however, a more accurate treatment of the transonic flow entering the nozzle exit cone would be required, as would consideration of boundary-layer effects in the nozzle.

The principle of "tuning" or adjusting nozzle design parameters to eliminate the effects of entering gas flow misalignment introduced by Darwell and Trubridge[1] can be extended to the case of misaligned nozzle exit cones, and substantial reductions in effective thrust misalignment can be achieved. The design freedom is somewhat more restricted than is the case for the Darwell-Trubridge situation, however, since the sinusoidal gain function has only one-half as many minimums as zeroes. It is clear that the methodology can be extended to the more general case in which the geometric misalignment is three dimensional. Results would have to be developed with the three-dimensional method of characteristics program for the yaw and roll planes.

There is the potential to enhance significantly the effectiveness of movable nozzle thrust vector control systems. Designs were found in this study in which the thrust vector was rotated some 30% more than the nozzle axis. However, it is not obvious that this degree of gain will be maintained at larger nozzle vector angles; a repetition of the analysis presented here at nozzle vector angles up to 15° would be required to explore this potential.

References

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Cost-Effective Radioisotope Thermoelectric Generator Designs Involving Cm-244 and Pu-238 Heat Sources

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This paper represents a comparative analysis of some of the technical considerations surrounding the use of Cm-244 and Pu-238 heat sources in radioisotope thermoelectric generators (RTGs). The principal considerations include radiological shielding, ground handling, and generator performance characteristics. The paper also describes a novel approach to RTG design and qualification which would facilitate the use of Cm-244 heat sources. This approach, which involves hermetically sealed bellows-encapsulated thermoelectric elements, also features the potential for increased generator output power stability and reliability and lower converter costs accruing from the advantages of a standardised approach to generator design, fabrication, and qualification.

Introduction

THE application of radioisotope thermoelectric generators (RTGs) to space electrical power requirements has been successfully demonstrated during the past five years with notable successes in the Apollo program (SNAP 27), the Nimbus III RTG Experiment (SNAP 19), and the ongoing Pioneer mission to Jupiter (SNAP 19). For these and similar missions, considerable attention has been focused on maximizing specific power [w (e)/kg] and minimizing degradation in output electrical power while maximizing energy conversion efficiency thereby minimizing thermal inventory. Consequently, the cost effectiveness of the RTG designs selected for previous extraterrestrial missions has generally been given less emphasis in an attempt to maximize the over-all RTG performance. However, the trend towards increasing numbers of military satellites (e.g., reconnaissance satellites) as well as remote terrestrial applications has prompted the reconsideration of RTG designs with a view towards bringing RTG performance into balance with RTG cost effectiveness.

The present paper describes the results of a study directed at identifying potentially cost-effective RTG designs involving Cm-244 and Pu-238 heat sources. The generator concept which evolved from this study involves hermetically sealed, bellows-encapsulated thermoelectric elements. This approach features the potential for greatly increased generator output power stability and reliability. The proposed RTG concept also provides for ease of heat-source replacement as well as generator repair in the event of premature failure of any of the individual thermoelectric couples. More significantly, this standardized approach lends itself to cost-effective methods of manufacture and assembly of RTG components and over-all systems.

A recent study[1,2] of availability, cost, and application of the Cm-244 radioisotope in RTG power systems enumerated the advantages of using this fuel in place of the presently used Pu-238 radioisotope fuel. It was concluded in the referenced study that curium will be available in large quantities in the form of reactor waste products—possibly beyond any foreseen demand for radioisotopic heat sources. The study also discussed the various alternatives for the recovery of curium from existing sources as well as suggesting several short-term and long-term options for increasing projected curium availability. The study concluded that production quantities of curium can be obtained at costs between \$20 and \$100 per thermal watt (w_{th}).

The reference study[2] also discussed the application of Cm-244 radioisotopic heat sources to the Multi-Hundred Watt (MHW) program. This study revealed that the total program costs (which include such cost elements as fuel, heat source fabrication, converter, ground support equipment, ground handling and acceptance tests, and management and radiological safety tests) for the Cm-244 RTGs was lower by a factor of 1.8 for the MHW converter. According to the study, the need for biological shielding increases the costs associated with fueling, testing, and transport of Cm-244 heat sources. This characteristic of Cm-244, together with its relatively short half life (18 yr as compared with 89 yr for Pu-238) presents some problems in its utilization

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